A NUMERICAL PROCEDURE FOR THE SOLUTION OF STRUCTURAL DYNAMICS PROBLEMS WITH CONCENTRATED MASSES

BY

ZUR-KONG LEE

B.S., St. John's University, 1938
M.S., University of Illinois, 1949

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
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Introduction

The object of the present investigation is to develop a new method for solving structural dynamics problems; a method which, with a reasonable amount of work, can give an approximate answer sufficiently accurate for ordinary engineering purposes and which, with additional work, yields results to any desired degree of accuracy. A numerical approach is chosen on account of its adaptability to the above requirements. The flexural vibration of beams due to impulsive or shock loads is taken up for study, but the principles can be extended to many other transient problems. Even though the title deals with concentrated masses, the application of the method is not so limited. The method is exact where masses are concentrated and it can approximate to any desired degree of accuracy the exact solution in cases where mass is distributed

A Numerical Procedure

The governing differential equations of the vibratory motion of a beam with concentrated masses consist of a system of simultaneous linear differential equations involving only the dependent variables, their time derivatives and the disturbance functions. A small time interval is chosen and the disturbance functions in that interval are represented by polynomials in the powers of time t, the degree of each polynomial being made as high as necessary to give the desired accuracy. A number of initial conditions are generally given such that together with the given differential equations and the equations obtained by differentiating the given equations successively with respect to time, all time derivatives of the dependent variables at the beginning of the interval can be

evaluated. Taylor's expansions in terms of the powers of the time interval are used to find the values of the dependent variables and their time derivatives at the end of the interval, the time derivatives at the beginning of the interval being the coefficients in the expansions. Only a sufficient number of time derivatives for each dependent variable need be found such that together with the value of the dependent variable itself, they serve as the initial conditions of the next interval, enabling all other time derivatives to be computed as has been done in the first interval. The process is repeated and this constitutes the step-by-step method.

For beams with distributed mass and flexibility, the governing equation of motion contains space as well as time derivatives of the dependent variable. The space derivatives are replaced by finite differences or by other linear relations in the dependent variable, which corresponds to replacing the given beam with one having concentrated masses. The method cited above then follows. This involves an approximation, but any desired accuracy can be attained by increasing the number of equivalent concentrations.

The procedure has general application to all cases in which the governing differential equation is linear, with constant coefficients, and in which all time derivatives at the initial instant can be evaluated by means of the initial conditions and the given differential equations. Various modifications based on the principle of superposition are used to save computational work. Extension of the procedure to cases where the coefficients are not constant is discussed. Treatment of different means of support, variation of section, axial loads and other special conditions is formulated.

Test Problems and Results

To test the effectiveness and limitations in the application of the procedure, four problems are solved by the step-by-step method.

(1) Vibration of beam with concentrated masses and flexi-

bility, ends simply supported, damping neglected.

(2) Vibration of beam with distributed mass and flexibility, ends simply supported, damping neglected.

(3) Vibration of beam with distributed mass and flexibility, ends simply supported, internal damping considered.

(4) Cantilever beam subjected to prescribed motion at the free end, damping neglected.

Comparison of the step-by-step solutions with exact solutions reveals that the step-by-step solutions are exact in cases with concentrated masses and that for cases where the mass is distributed, the use of only a few equivalent concentrations to approximate the actual distribution of mass is surprisingly satisfactory. Sufficient evidence has been obtained to demonstrate the suitability of the method for studying the motion of actual structures.

Concluding Remarks

The present investigation has developed a method which can be used to solve many structural dynamics problems of a practical nature. It remains to utilize it in making exhaustive studies of unsolved specific problems whose solutions may contribute to the advancement of engineering knowledge.

VITA

Zur-kong Lee was born in Shanghai, China, on August 30, 1917. He graduated from St. John's University, Shanghai, China, with the degree of Bachelor of Science (with honor) in Civil Engineering in June 1938. His practical experience consists in designing buildings, runway, wharves and water towers during the periods from July 1938 to August 1941 and from January 1946 to August 1947 while he was connected with "The Southeastern Company, Architects and Engineers" and "Kwan, Chu and Yang, Architects and Engineers". He taught in the Department of Civil Engineering of his Alma Mater during the period from September 1941 to June 1947.

He began his graduate work at the University of Illinois in September 1947 and received the degree of Master of Science in February 1949. He held a Research Assistantship from September 1947 to June 1949. In July 1949, he was appointed Special Research Associate in Civil Engineering, which position he now holds.

He was elected a member of Phi Tau Phi National Scholastic Honors Society of China in April 1941 and an associate member of Sigma Xi in April 1948.